දකුණු පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

13 ශ්‍රේණිය - අවසාන වාර පරීක්ෂණය - 2025

සංයුක්ත ගණිතය - I

(01)

$$S_r = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r}$$

$$S_1 + S_2 + \dots + S_n = (n+1)S_n - n$$

$L:H:S = S_1$
$= 1$

$R:H:S = (1+1)S_1 - 1$
$= 2 \times 1 - 1$
$= 1$

$L:H:S = R:H:S$ 5

$\therefore$ $n = 1$ ට ප්‍රතිඵලය සත්‍ය වේ.

$n = p$ ට ප්‍රතිඵලය සත්‍ය යයි උපකල්පනය කරමු.

$$S_1 + S_2 + S_3 + \dots + S_p = (p+1)S_p - p \quad 5$$

$n = p+1$ විට

$$S_1 + S_2 + S_3 + \dots + S_p + S_{p+1} = (p+1)S_p - p + S_{p+1} \text{ — } ①$$

මෙහි,

$$S_p = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{p}$$

$$S_{p+1} = S_p + \frac{1}{p+1} \quad 5$$

$$\therefore (p+1)S_{p+1} = (p+1)S_p + 1$$

$$(p+1)S_p = (p+1)S_{p+1} - 1$$

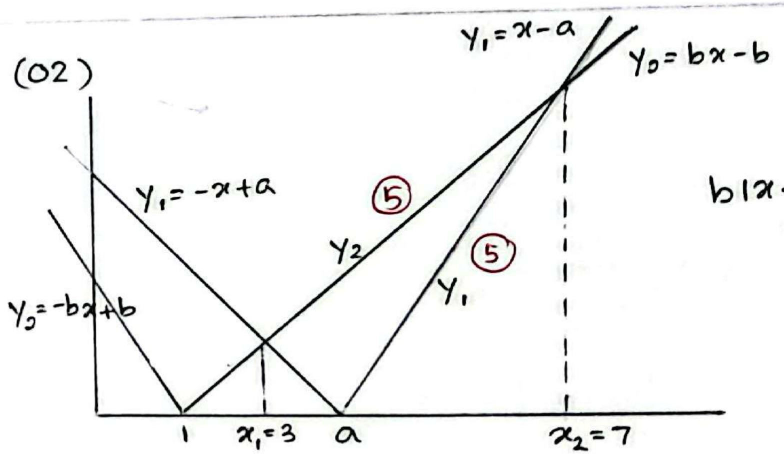
① න්,

$$S_1 + S_2 + S_3 + \dots + S_p + S_{p+1} = (p+1)S_{p+1} - 1 - p + S_{p+1} \quad 5$$
$$= (p+2)S_{p+1} - (p+1)$$
$$= (\overline{p+1}+1)S_{\overline{p+1}} - (\overline{p+1})$$

$\therefore$ $n = p+1$ ට ද ප්‍රතිඵලය සත්‍ය වේ. $n = p$ සඳහා ප්‍රතිඵලය සත්‍ය නම් $n = p+1$ සඳහා ද ප්‍රතිඵලය සත්‍ය වේ. $n = 1$ සඳහා ප්‍රතිඵලය සත්‍ය බැවින් ගණිත අභ්‍යුහන මූලධර්මය මගින් සියලු $n \in \mathbb{Z}^+$ සඳහා ප්‍රතිඵලය සත්‍ය වේ. 5

$\boxed{25}$

(02)



$$b|x-1| > |x-a|$$
$$y_2 > y_1$$

$$-x_1 + a = bx_1 - b$$

$x_1 = 3$ විට,

$$-3 + a = 3b - b$$
$$2b - a = -3 \quad 5 \text{ — } ①$$

$$x_2 - a = bx_2 - b$$

$x_2 = 7$ විට

$$7 - a = 7b - b$$
$$6b + a = 7 \quad 5 \text{ — } ②$$

$① + ② \Rightarrow$ $8b = 4$

$\underline{\underline{b = 1/2}}$ 5

$① \Rightarrow$ $1 - a = -3$

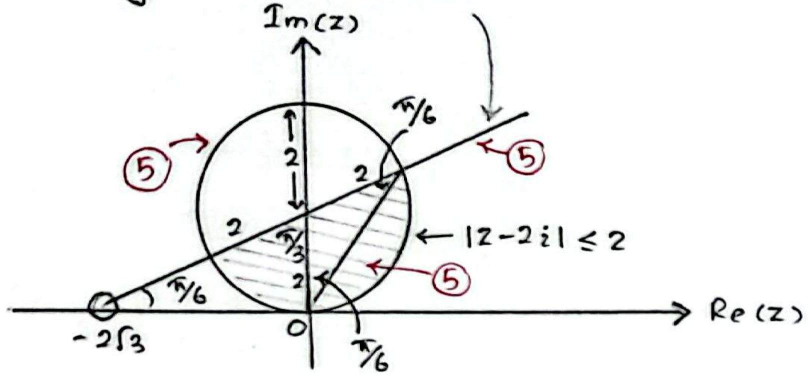
$\underline{\underline{a = 4}}$

$\boxed{25}$

(03) $|\overline{Z}+2i| = |\overline{\overline{Z}+2i}|$

$= |Z-2i| \leq 2$

$0 \leq Arg(Z+2\sqrt{3}) \leq \pi/6$



$|Z|$ හි වැඩිතම අගය $= 2\cos\pi/6 + 2\cos\pi/6$ 5

$= 4\cos\pi/6$

$= 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$ 5

$\boxed{25}$

(04) $\left(3\sqrt{2}x - \frac{a}{\sqrt{2}}\right)^8 = \frac{1}{16}$

5 $x=1$ විට ; $\left(3\sqrt{2} - \frac{a}{\sqrt{2}}\right)^8 = \left(\frac{1}{\sqrt{2}}\right)^8$ 5

$\left(\frac{6-a}{\sqrt{2}}\right)^8 = \left(\frac{1}{\sqrt{2}}\right)^8$

$\frac{6-a}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}} \Rightarrow 6-a = \pm 1$ 5

5 $a = 5$ හෝ $a = 7$ 5 $\boxed{25}$

(05) $\lim_{x\to 1} \frac{(x^{1/4}+2x^{1/3}-3)\sin(x-1)}{(x-1)^2}$

$= \lim_{x\to 1} \frac{x^{1/4}-1^{1/4}}{x-1} + 2\lim_{x\to 1}\frac{x^{1/3}-1^{1/3}}{x-1} + \lim_{x\to 1}\frac{\sin(x-1)}{(x-1)}$ 5 5 5

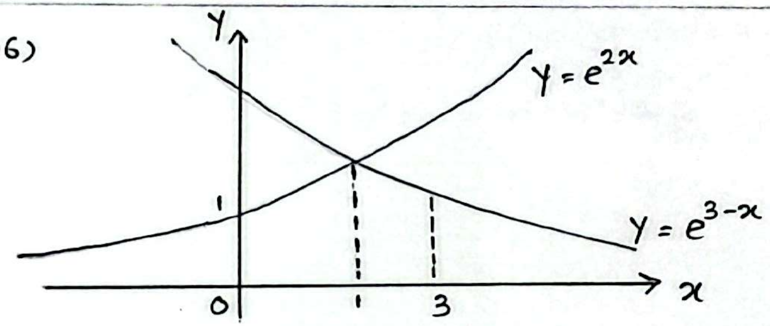
$= \left[\frac{1}{4}(1)^{-3/4} + 2\times\frac{1}{3}(1)^{-2/3}\right] \times 1$ 5

$= \left(\frac{1}{4}+\frac{2}{3}\right)\times 1$

$= \frac{11}{12}$ 5

$\boxed{25}$

(06)



$e^{2x} = e^{3-x}$

$2x = 3-x$

$x = 1$ 5

$V = \int_0^1 \pi(e^{2x})^2 dx + \int_1^3 \pi(e^{3-x})^2 dx$ 5 5

$= \pi\left[\int_0^1 e^{4x}dx + \int_1^3 e^{6-2x}dx\right]$

$= \pi\left\{\frac{\left[e^{4x}\right]_0^1}{4} + \frac{\left[e^{6-2x}\right]_1^3}{-2}\right\}$ 5

$= \pi\left[\frac{e^4-e^0}{4} - \frac{1}{2}(e^0-e^4)\right] = \pi/4\,(3e^4-3)$ 5

$= \frac{3\pi}{4}(e^4-1)$

$\boxed{25}$

(07)

$$x = \frac{a}{2}\left(t+\frac{1}{t}\right) \quad , \quad y = \frac{b}{2}\left(t-\frac{1}{t}\right)$$

$$\frac{dx}{dt} = \frac{a}{2}\left(1-\frac{1}{t^2}\right) \text{ (5)}, \quad \frac{dy}{dt} = \frac{b}{2}\left(1+\frac{1}{t^2}\right) \text{ (5)}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \cancel{\frac{b}{2}\left(1+\frac{1}{t^2}\right) \times \frac{2}{a}\frac{t^2}{(t^2-1)}}$$

$$= \frac{b}{2}\left(\frac{t^2+1}{t^2}\right) \times \frac{2}{a}\left(\frac{t^2}{t^2-1}\right)$$

$$E = \frac{dy}{dx} = \frac{b}{a}\frac{(t^2+1)}{(t^2-1)} \text{ (5)}$$

$$aEt^2 - aE = bt^2 + b$$

$$(aE-b)t^2 = b + aE$$

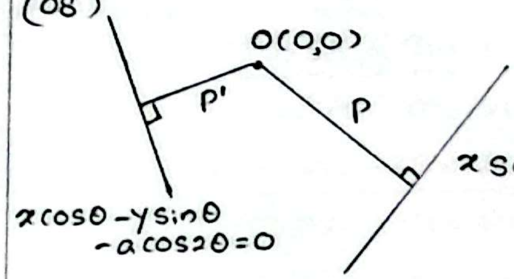
$$t^2 = \frac{b+aE}{aE-b} > 0 \text{ (5)}$$ විය යුතු බැවින්

$E < -b/a$ හෝ $E > b/a$ විය යුතුය (5)

$\therefore$ වක්‍රයේ අනුක්‍රමණය $\left(E = \frac{dy}{dx}\right)$ $-b/a$ සහ b/a අතර නොපවතී.

25

(08)



$$p = \frac{|-a|}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}} \text{ (5)}$$

$$p' = \frac{|-a\cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} \text{ (5)}$$

$$a^2 = p^2\left(\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}\right)$$

$$(p')^2 = a^2\cos^2 2\theta \text{ (5)}$$

$$a^2 = \frac{p^2(\sin^2\theta + \cos^2\theta)}{\sin^2\theta \cdot \cos^2\theta}$$

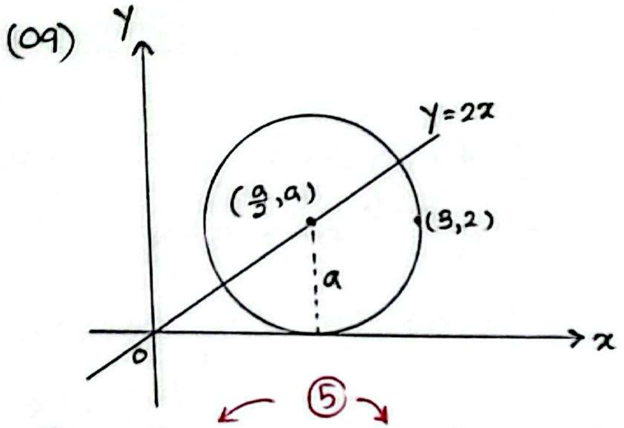
$$p^2 = \frac{a^2\sin^2 2\theta}{4} \text{ (5)}$$

$$4p^2 + (p')^2 = a^2\sin^2 2\theta + a^2\cos^2 2\theta \text{ (5)}$$

$$4p^2 + (p')^2 = a^2$$

25

(09)



කේන්ද්‍රය $(\frac{a}{2}, a)$ ද අරය a ද යැයි ගනිමු. 5

$$\left(\frac{a}{2}-3\right)^2+(a-2)^2=a^2 \quad 5$$

$$\frac{a^2}{4}-3a+9+a^2-4a+4=a^2$$

$$\frac{a^2}{4}-7a+13=0$$

$$a^2-28a+52=0$$

$$(a-26)(a-2)=0 \quad 5$$

$a=26$ හෝ $a=2$

∴ වෘත්ත දෙක

$$(x-13)^2+(y-26)^2=26^2 \quad 5$$

සහ

$$(x-1)^2+(y-2)^2=2^2 \quad 5$$

$\boxed{25}$

(10)

$$L.H.S = \frac{\sin 3A+\sin^3 A}{\cos^3 A-\cos 3A}$$

$$= \frac{3\sin A-4\sin^3 A+\sin^3 A}{\cos^3 A-(4\cos^3 A-3\cos A)}$$

$$= \frac{3\sin A-3\sin^3 A}{3\cos A-3\cos^3 A} \quad 5 \quad 5$$

$$= \frac{3\sin A(1-\sin^2 A)}{3\cos A(1-\cos^2 A)} = \frac{\cancel{\sin A}\cdot\cos^2 A}{\cancel{\cos A}\cdot\sin^2 A} \quad 5$$

$$= \cot A = R.H.S.$$

$$\frac{\sin 3x+\sin^3 x}{\cos^3 x-\cos 3x}=1$$

$$\cot x = 1 \quad 5$$

$$\tan x = 1$$

$$\tan x = \tan \pi/4$$

$$x = n\pi+\pi/4 \quad 5 \quad ; n\in\mathbb{Z}$$

$\boxed{25}$

B කොටස

(11)

(a) $f(x) = x^2 + ax + b$ ⟨ α, β

$\alpha + \beta = -a$ (5) $\alpha\beta = b$

$\alpha^2 - 1$ සහ $\beta^2 - 1$ මූල වන වර්ගජ සමීකරණය

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (5)
$= (-a)^2 - 2b$
$= a^2 - 2b$ (5)

$$x^2 - (\alpha^2 - 1 + \beta^2 - 1)x + (\alpha^2 - 1)(\beta^2 - 1) = 0 \quad (5)$$

$$x^2 - (\alpha^2 + \beta^2 - 2)x + \alpha^2\beta^2 - (\alpha^2 + \beta^2) + 1 = 0$$

$$x^2 - (a^2 - 2b - 2)x + b^2 - (a^2 - 2b) + 1 = 0 \quad (5)\ (5)$$

$$x^2 - (a^2 - 2b - 2)x + b^2 - a^2 - 2b + 1 = 0$$

$\boxed{30}$

$x = \alpha^2 - 1,\ \beta^2 - 1$

$y = \alpha^2(\beta^2 - 1),\ \beta^2(\alpha^2 - 1)$

$y = \alpha^2\beta^2 - \alpha^2,\ \alpha^2\beta^2 - \beta^2$ (5)

$y = b^2 - (x + 1)$ (10)

$x = b^2 - 1 - y$

$\therefore$ $\alpha^2(\beta^2 - 1)$ සහ $\beta^2(\alpha^2 - 1)$ මූල වන වර්ගජ සමීකරණය

$\boxed{25}$

(10)

$$(b^2 - 1 - y)^2 - (a^2 - 2b - 2)(b^2 - 1 - y) + b^2 - a^2 + 2b + 1 = 0$$

$$y^2 - 2(b^2 - 1)y + (b^2 - 1)^2 - (a^2 - 2b - 2)(b^2 - 1) + (a^2 - 2b - 2)y + b^2 - a^2 + 2b + 1 = 0$$

$$y^2 + (a^2 - 2b - 2 - 2b^2 + 2)y + (b^2 - 1)^2 - (a^2 - 2b - 2)(b^2 - 1) + b^2 - a^2 + 2b + 1 = 0$$

$$y^2 + (a^2 - 2b - 2b^2)y + (b^2 - 1)^2 - (b^2 - 1)^2 - (a^2 - 2b - 2)(b^2 - 1) + b^2 - a^2 + 2b + 1 = 0$$

(b) $E = x^4 - 4x^3 + 9x^2 - 10x + 7$

$E = (x^2 + bx + c)^2 + (x^2 + bx + c) + a$ (5)

$E = x^4 + b^2x^2 + c^2 + 2bx^3 + 2cx^2 + 2bcx + x^2 + bx + c + a$ (5)

$E = x^4 + 2bx^3 + (b^2 + 2c + 1)x^2 + (2bc + b)x + a + c + c^2$

$((x^3))$: $-4 = 2b \Rightarrow b = -2$ (5)

$((x^2))$: $b^2 + 2c + 1 = 9 \Rightarrow 4 + 2c + 1 = 9 \Rightarrow c = 2$ (5)

$((x))$: $2bc + b = 2(-2)(2) - 2 = -10$

$((x^0))$: $a + c + c^2 = 7 \Rightarrow a + 2 + 4 = 7 \Rightarrow a = 1$ (5)

$\therefore$ මෙය $E = y^2 + y + a$ ආකාරයෙන් ලිවිය හැක.

මෙහි $y = x^2 - 2x + 2$ (5)

එවිට $E = (x^2 - 2x + 2)^2 + (x^2 - 2x + 2) + 1$ (5)

$$E = \underbrace{[(x-1)^2 + 1]^2}_{\geq 1} + \underbrace{(x-1)^2 + 1}_{\geq 1} + 1 \quad (5)$$

$\therefore$ $E \geq 3$ (5)

$\boxed{45}$

(e) $f(x) = x^4+2x^3+2x^2-2x-3$

$f(1) = 1+2+2-2-3 = 0$ (5)

$\therefore (x-1)$, $f(x)$ හි සාධකයකි (5)

$f(-1) = 1-2+2+2-3 = 0$ (5)

$\therefore (x+1)$, $f(x)$ හි සාධකයකි (5)

$f(x) = \underbrace{(x-1)(x+1)}_{x^2-1}(x^2+2x+3)$ (5)

$x^4+2x^3-x^2-5x+7 = \underbrace{x^4+2x^3+2x^2-2x-3}\ -3x^2-3x+10$ (5) (5)

$= (x-1)(x+1)(x^2+2x+3) - 3(x^2+2x+3) + 3x+19$ (5) (5)

$= (x^2+2x+3)(x^2-1-3) + 3x+19$

$\therefore$ ශේෂය $= 3x+19$ (5)

$\boxed{50}$

(12)

(a)(i) E _ _ _ _ _ Y

මැද 5 දෙනා සඳහා ආකාර $= 5!$ (5)

E හා Y සඳහා ආකාර $= 2!$ (5)

$\therefore$ අදාළ වෙනස් ආකාර $= 5! \times 2! = 120 \times 2$

$= 240$ (5)

(ii) _ EY _ _ _ _

$\boxed{EY}$ එක ළඟ ඇති විට $= 6! \times 2!$ (5) (5)

$= 720 \times 2$

$= 1440$ (5)

(iii)

		E හා Y හැර අනෙක් ආකාර		Y සඳහා ආකාර
_ _ _ _ Y _ E	$\Rightarrow$	$5!$	$\times$	6
_ _ Y _ _ E _	$\Rightarrow$	$5!$	$\times$	5
_ Y _ _ E _ _	$\Rightarrow$	$5!$	$\times$	4
_ _ _ E _ _ _	$\Rightarrow$	$5!$	$\times$	3
_ _ E _ _ _ _	$\Rightarrow$	$5!$	$\times$	2
_ E _ _ _ _ _	$\Rightarrow$	$5!$	$\times$	1

Y හැම විටම E ට පසු සිටින විට ආකාර $\Big\} = 5!(6+5+4+3+2+1)$ (10)

$= 5! \times 21$

$= 2520$ (5)

$\boxed{45}$

(b) $\dfrac{3r+1}{(r+1)(r+2)(r+3)} = \dfrac{A}{(r+1)} + \dfrac{B}{(r+2)} + \dfrac{C}{(r+3)}$ (5)

$3r+1 = A(r^2+5r+6) + B(r^2+4r+3) + C(r^2+3r+2)$ (5)

$r^2 : \Rightarrow A+B+C = 0$ —— (1)

$r : \Rightarrow 5A+4B+3C = 3$ —— (2)

$r^0 : \Rightarrow 6A+3B+2C = 1$ —— (3)

(1) + (3) $\Rightarrow 7A+4B+3C = 1$ —— (4)

(4) − (2) $\Rightarrow 2A = -2 \Rightarrow A = -1$ (5)

$B+C=1$

$4B+3C=8$

$3B+3C=3$

$B=5$ (5) , $C=-4$ (5)

$\dfrac{3r+1}{(r+1)(r+2)(r+3)} = \dfrac{-1}{(r+1)} + \dfrac{5}{(r+2)} - \dfrac{4}{(r+3)}$ (5) $\boxed{30}$

$U_r = \dfrac{-1}{(r+1)} + \dfrac{1}{(r+2)} + \dfrac{4}{(r+2)} - \dfrac{4}{(r+3)}$ (5)

$= -\left[\dfrac{1}{r+1} - \dfrac{1}{r+2}\right] + 4\left[\dfrac{1}{r+2} - \dfrac{1}{r+3}\right]$

$\therefore f(r) = \dfrac{1}{r+1}$ (5) ද $\lambda = -1$ (5) ද $\mu = 4$ (5) ද වේ.

$U_r = -1[f(r) - f(r+1)] + 4[f(r+1) - f(r+2)]$

$r=1 ; \; U_1 = -1[f(1) - f(2)] + 4[f(2) - f(3)]$

$r=2 ; \; U_2 = -1[f(2) - f(3)] + 4[f(3) - f(4)]$ (5)

$r=3 ; \; U_3 = -1[f(3) - f(4)] + 4[f(4) - f(5)]$

$\vdots$

$r=(n-2) ; \; U_{n-2} = -1[f(n-2) - f(n-1)] + 4[f(n-1) - f(n)]$

$r=(n-1) ; \; U_{n-1} = -1[f(n-1) - f(n)] + 4[f(n) - f(n+1)]$ (5)

$r=n ; \; U_n = -1[f(n) - f(n+1)] + 4[f(n+1) - f(n+2)]$

+

$\sum_{r=1}^{n} U_r = -1[f(1) - f(n+1)] + 4[f(2) - f(n+2)]$ (5)

$= -1\left[\dfrac{1}{2} - \dfrac{1}{n+2}\right] + 4\left[\dfrac{1}{3} - \dfrac{1}{n+3}\right]$ (5)

$= -\dfrac{1}{2} + \dfrac{4}{3} + \dfrac{1}{n+2} - \dfrac{4}{n+3}$

$= \dfrac{-3+8}{6} + \dfrac{n+3-4(n+2)}{(n+2)(n+3)}$

$= \dfrac{5}{6} - \dfrac{(3n+5)}{(n+2)(n+3)}$ (5) $\boxed{45}$

$\lim_{n\to\infty}\left(\sum_{r=1}^{n} U_r\right) = \lim_{n\to\infty} \dfrac{5}{6} - \dfrac{(3n+5)}{(n+2)(n+3)}$

$= \lim_{n\to\infty} \dfrac{5}{6} - \dfrac{n(3+5/n)}{n^2(1+2/n)(1+3/n)}$ (5)

$= 5/6$ (5)

∴ මෙය පරිමිත අගයකි . ∴ අපරිමිත ශ්‍රේණිය අභිසාරී වේ. (5)

(b) $\dfrac{3r+1}{(r+1)(r+2)(r+3)} = \dfrac{A}{(r+1)} + \dfrac{B}{(r+2)} + \dfrac{C}{(r+3)}$ (5)

$3r+1 = A(r^2+5r+6) + B(r^2+4r+3) + C(r^2+3r+2)$ (5)

$r^2 : \Rightarrow A+B+C = 0$ —— (1)

$r : \Rightarrow 5A+4B+3C = 3$ —— (2)

$r^0 : \Rightarrow 6A+3B+2C = 1$ —— (3)

(1) + (3) $\Rightarrow 7A+4B+3C = 1$ —— (4)

(4) − (2) $\Rightarrow 2A = -2 \Rightarrow A = -1$ (5)

$B+C = 1$

$4B+3C = 8$

$3B+3C = 3$

$B = 5$ (5) , $C = -4$ (5)

$\dfrac{3r+1}{(r+1)(r+2)(r+3)} = \dfrac{-1}{(r+1)} + \dfrac{5}{(r+2)} - \dfrac{4}{(r+3)}$ (5) $\boxed{30}$

$U_r = \dfrac{-1}{(r+1)} + \dfrac{1}{(r+2)} + \dfrac{4}{(r+2)} - \dfrac{4}{(r+3)}$ (5)

$= -\left[\dfrac{1}{r+1} - \dfrac{1}{r+2}\right] + 4\left[\dfrac{1}{r+2} - \dfrac{1}{r+3}\right]$

$\therefore f(r) = \dfrac{1}{r+1}$ (5) ද $\lambda = -1$ (5) ද $\mu = 4$ (5) ද වේ.

$U_r = -1[f(r) - f(r+1)] + 4[f(r+1) - f(r+2)]$

$r=1 \; ; \; U_1 = -1[f(1) - f(2)] + 4[f(2) - f(3)]$

$r=2 \; ; \; U_2 = -1[f(2) - f(3)] + 4[f(3) - f(4)]$ (5)

$r=3 \; ; \; U_3 = -1[f(3) - f(4)] + 4[f(4) - f(5)]$

$\vdots$

$r=(n-2) \; ; \; U_{n-2} = -1[f(n-2) - f(n-1)] + 4[f(n-1) - f(n)]$

$r=(n-1) \; ; \; U_{n-1} = -1[f(n-1) - f(n)] + 4[f(n) - f(n+1)]$ (5)

$r=n \; ; \; U_n = -1[f(n) - f(n+1)] + 4[f(n+1) - f(n+2)]$

+

$\displaystyle\sum_{r=1}^{n} U_r = -1[f(1) - f(n+1)] + 4[f(2) - f(n+2)]$ (5)

$= -1\left[\dfrac{1}{2} - \dfrac{1}{n+2}\right] + 4\left[\dfrac{1}{3} - \dfrac{1}{n+3}\right]$ (5)

$= \dfrac{-1}{2} + \dfrac{4}{3} + \dfrac{1}{n+2} - \dfrac{4}{n+3}$

$= \dfrac{-3+8}{6} + \dfrac{n+3-4(n+2)}{(n+2)(n+3)}$

$= \dfrac{5}{6} - \dfrac{(3n+5)}{(n+2)(n+3)}$ (5) $\boxed{45}$

$\displaystyle\lim_{n\to\infty}\left(\sum_{r=1}^{n} U_r\right) = \lim_{n\to\infty} \dfrac{5}{6} - \dfrac{(3n+5)}{(n+2)(n+3)}$

$= \displaystyle\lim_{n\to\infty} \dfrac{5}{6} - \dfrac{n(3+5/n)}{n^2(1+2/n)(1+3/n)}$ (5)

$= 5/6$ (5)

∴ මෙය පරිමිත අගයකි. ∴ අපරිමිත ශ්‍රේණිය අභිසාරී වේ. (5)

$$\sum_{r=1}^{1} U_r = U_1 = \frac{(3+1)}{(2)(3)(4)} \qquad \sum_{r=1}^{\infty} U_r = \frac{5}{6} \; 5$$

$$U_1 = \frac{1}{6} \; 5$$

$$\frac{1}{6} \le \sum_{r=1}^{n} U_r < \frac{5}{6} \Rightarrow \frac{1}{6} \le \frac{5}{6} - \frac{(3n+5)}{(n+2)(n+3)} < \frac{5}{6} \; 5$$

$\boxed{30}$

---

(13) (a) $A = \begin{pmatrix} 3 & p \\ -2 & -3 \end{pmatrix}$

$$\det A = \begin{vmatrix} 3 & p \\ -2 & -3 \end{vmatrix} = -9+2p \; 5$$

A^{-1} පැවතීම සඳහා $-9+2p \neq 0$ විය යුතුය.

$$p \neq 9/2 \; 5$$

$$A^{-1} = A$$

$$\frac{1}{(2p-9)} \begin{pmatrix} -3 & -p \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & p \\ -2 & -3 \end{pmatrix} \; 5$$

$$\frac{-3}{2p-9} = 3 \quad , \quad \frac{-p}{2p-9} = p$$

$$\frac{2}{2p-9} = -2 \quad , \quad \frac{3}{2p-9} = -3$$

මෙයින් ඕනෑම එකක් සඳහා 5

$\therefore 2p-9 = -1$ හා $p(2p-9+1) = 0$

$2p = 8$ $\qquad$ $p \neq 0$ බැවින් $2p-8 = 0$

$p = 4$ 5 $\qquad$ $p = 4$

$$A^{-1} = A$$

$$AA^{-1} = AA \; 5$$

$$I = A^2$$

$$A^2 - I = 0$$

$$(A-I)(A+I) = 0 \; 5$$

මෙය $BC = 0$ ආකාර ගනී.

$$A - I = \begin{pmatrix} 3 & 4 \\ -2 & -3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 4 \\ -2 & -4 \end{pmatrix} \; 5$$

හා

$$A + I = \begin{pmatrix} 3 & 4 \\ -2 & -3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 4 & 4 \\ -2 & -2 \end{pmatrix} \; 5$$

$\boxed{45}$

(b) $Z = x + iy$ යැයි ගනිමු.

$$Z\bar{Z} = (x+iy)(x-iy) \; 5$$

$$= x^2 - i^2 y^2$$

$$= x^2 + y^2 \; 5$$

$$= |Z|^2$$

$\boxed{10}$

CamScanner

$\left|\frac{z}{w}-1\right|^2 = \left(\frac{z}{w}-1\right)\overline{\left(\frac{z}{w}-1\right)}$ 5

$= \left(\frac{z}{w}-1\right)\left[\overline{\left(\frac{z}{w}\right)}-\bar{1}\right]$ 5

$= \frac{z}{w}\cdot\frac{\bar{z}}{\bar{w}} - \frac{z}{w} - \frac{\bar{z}}{\bar{w}} + 1$

$= 1 + \left|\frac{z}{w}\right|^2 - \left[\frac{z}{w} + \overline{\left(\frac{z}{w}\right)}\right]$ 5 $\frac{z+\bar{z}}{2} = Re(z)$ බැවින්

$\left|\frac{z}{w}-1\right|^2 = 1 + \left|\frac{z}{w}\right|^2 - 2Re\left(\frac{z}{w}\right)$ 5

මෙලෙසම $\left|\frac{z}{w}+1\right|^2 = 1+\left|\frac{z}{w}\right|^2 + 2Re\left(\frac{z}{w}\right)$ 5

$\boxed{25}$

$\therefore \left|\frac{z}{w}-1\right|^2 = \left|\frac{z-w}{w}\right|^2 = 1+\left|\frac{z}{w}\right|^2 - 2Re\left(\frac{z}{w}\right)$

5

$|z-w|^2 = |w|^2+|z|^2-2|w|^2 Re\left(\frac{z}{w}\right)$ —— (1)

$\left|\frac{z}{w}+1\right|^2 = \left|\frac{z+w}{w}\right|^2 = 1+\left|\frac{z}{w}\right|^2 + 2Re\left(\frac{z}{w}\right)$

5

$|z+w|^2 = |w|^2+|z|^2+2|w|^2 Re\left(\frac{z}{w}\right)$ —— (2)

$|z+w| = |z-w|$ බැවින්

5

$|w|^2+|z|^2-2|w|^2 Re\left(\frac{z}{w}\right) = |w|^2+|z|^2+2|w|^2 Re\left(\frac{z}{w}\right)$

$4|w|^2 Re\left(\frac{z}{w}\right) = 0 \Rightarrow Re\left(\frac{z}{w}\right) = 0$

5

$w \neq 0$ බැවින්

$\boxed{20}$

(2) නම්

$|z-w|^2 = |w|^2+|z|^2$ 5

$|z| = k|w|$

$\frac{|z|}{|w|} = k \Rightarrow \left|\frac{z}{w}\right| = k$ 5

$Re\left(\frac{z}{w}\right) = 0$ බැවින් 5

$\therefore \frac{z}{w} = ki$ 5

$z = kwi$

$\boxed{20}$

(c) $4\cos\frac{\pi}{12}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$= 4\cos^2\frac{\pi}{12} + 4i\sin\frac{\pi}{12}\cos\frac{\pi}{12}$ 5

$= 2\left(1+\cos\frac{\pi}{6}\right) + 2i\sin\frac{\pi}{6}$ 5

$= 2\left(1+\frac{\sqrt{3}}{2}\right) + 2i\times\frac{1}{2}$

$= 2+\sqrt{3}+i$ 5

$(2+\sqrt{3}+i)^6 = \left(4\cos\frac{\pi}{12}\right)^6\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)^6$ 5

$= 2^{12}\cos^6\frac{\pi}{12}\left[\cos\frac{6\pi}{12} + i\sin\frac{6\pi}{12}\right]$ 5

$= 2^{12}\cos^6\frac{\pi}{12}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ $\cancel{5}$

$= 2^{12}\cos^6\frac{\pi}{12}(0+i)$ 5

$= 2^{12}\cdot i\cdot\cos^6\frac{\pi}{12}$

$= 2^{12}\left[\cos^6\left(\frac{\pi}{12}\right)\right]i$

$\boxed{35}$

(14) $f(x) = \dfrac{3-4x}{(x-1)(x-3)}$ (5)

$f'(x) = \dfrac{(x-1)(x-3)(-4) - (3-4x)(x-1+x-3)}{(x-1)^2(x-3)^2}$ (5)

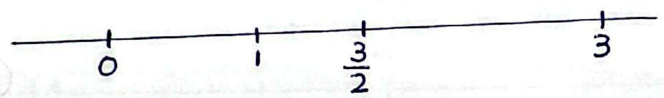
$= \dfrac{-4\,[x^2-4x+3] - (3-4x)(2x-4)}{(x-1)^2(x-3)^2}$

$= \dfrac{-(-4x^2+6x)}{(x-1)^2(x-3)^2}$ (5) $\boxed{15}$

$f'(x) = \dfrac{2x(2x-3)}{(x-1)^2(x-3)^2}$

$f'(x) = 0 \Leftrightarrow 2x(2x-3) = 0$ (5)

$x = 0$ හෝ $x = \frac{3}{2}$



	$x<0$	$0<x<1$	$1<x<\frac{3}{2}$	$\frac{3}{2}<x<3$	$x>3$
$f'(x)$	+ ↗	− ↘	− ↘	+ ↗	+ ↗

(15)

f වැඩිවන ප්‍රාන්තර $(-\infty, 0] \cup [\frac{3}{2}, 3) \cup (3, \infty)$ (5)

f අඩුවන ප්‍රාන්තර $[0,1) \cup (1, \frac{3}{2}]$ (5)

$\boxed{30}$

$y = \dfrac{3-4x}{(x-1)(x-3)}$

$x = 0 \quad y = \dfrac{3}{(-1)(-3)} = 1$

$x = \frac{3}{2} \quad y = \dfrac{3 - 4\times\frac{3}{2}}{(\frac{3}{2}-1)(\frac{3}{2}-3)} = \dfrac{3-6}{\frac{1}{2}\times\frac{-3}{2}} = 4$

$(0, 1)$ හිදී උපරිමයක් වේ. 5

$(\frac{3}{2}, 4)$ හිදී අවමයක් වේ. 5

සිරස් ස්පර්ශෝන්මුඛ

$$(x-1)(x-3) = 0$$

$x = 1$, $x = 3$ සිරස් ස්පර්ශෝන්මුඛ වේ. 5

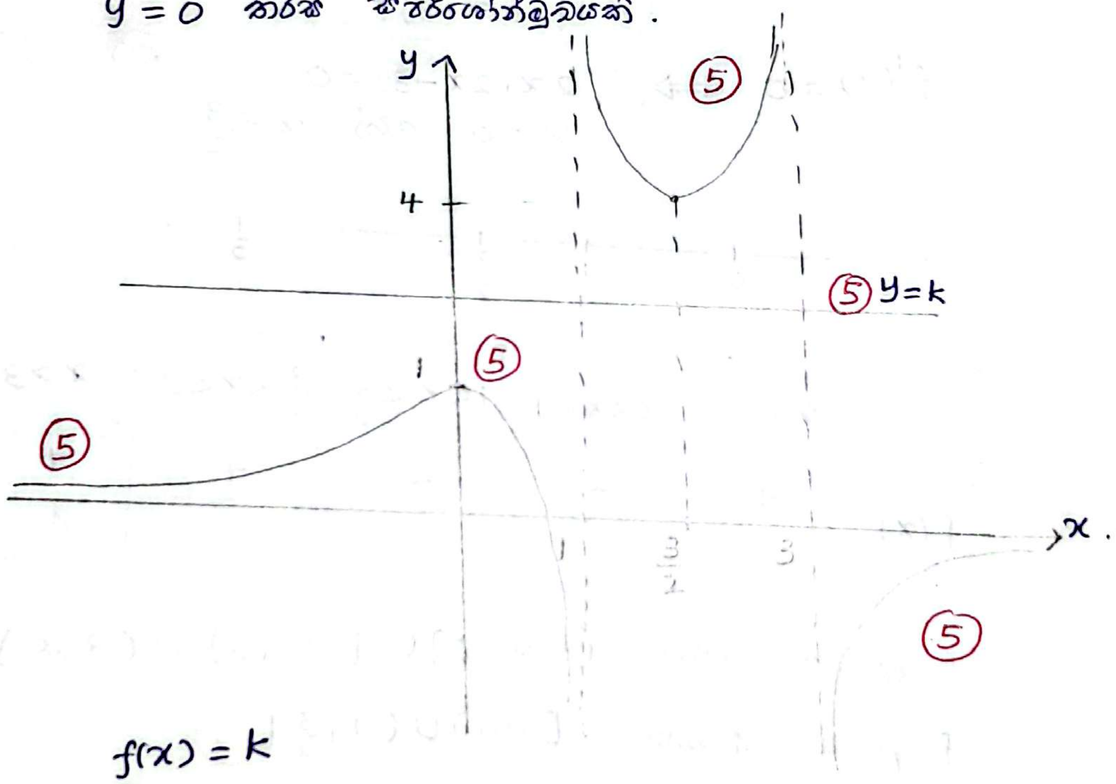
$x \rightarrow 1^+$ $\quad y \rightarrow \infty$ $\quad x \rightarrow 3^+$ $\quad y \rightarrow -\infty$

$x \rightarrow 1^-$ $\quad y \rightarrow -\infty$ $\quad x \rightarrow 3^-$ $\quad y \rightarrow +\infty$

$$\lim_{x \rightarrow \pm\infty} y \rightarrow 0$$

5

$y = 0$ තිරස් ස්පර්ශෝන්මුඛයකි.



$f(x) = k$

* $k > 4$, $0 < k < 1$ හෝ $k < 0$ විට $y = k$ රේඛාව $f(x)$ වක්‍රය ප්‍රභින්න ලක්ෂ්‍ය දෙකක් ඡේදනය කරයි.

$k > 4$ හෝ $0 < k < 1$ හෝ $k < 0$. 10

* $1 < k < 4$ විට $y = k$ රේඛාව වක්‍රය ඡේදනය නොකරයි.

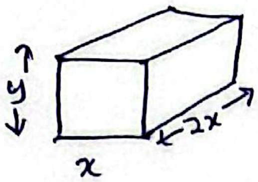
$\therefore$ විසඳුම් නොතිබෙන පරිදි k හි පරාසය $1 < k < 4$ වේ.

5

60

14

(b)



$$V = 36$$

$$2x^2y = 36$$

$$y = \frac{18}{x^2} \quad (5)$$

පෘෂ්ඨ වර්ගඵලය $A = 2x^2 + 2(2xy) + 2(xy)$ (5)

$$A = 2x^2 + 6xy$$

$$A = 2x^2 + 6x \times \frac{18}{x^2}$$

$$A = 2x^2 + \frac{108}{x} \quad (5)$$

$$\frac{dA}{dx} = 4x - \frac{108}{x^2} = \frac{4(x^3 - 27)}{x^2} \quad (10)$$

$$\frac{dA}{dx} = 4(x-3)\frac{(x^2+3x+9)}{x^2}$$

$\frac{dA}{dx} = 0$ විට $x = 3$ (5)

$0 < x < 3$ විට $\frac{dA}{dx} < 0$ (5)

$x > 3$ විට $\frac{dA}{dx} > 0$ (5)

$\therefore x = 3$ විට A අවම වේ. (5)

45

(15) (a) $2x^2+3x+1 = A(x-1)(x^2+2) + B(x^2+2) - (x+C)(x-1)^2$

$x=1, \quad 2+3+1 = 0+3B-0$ 5

$\therefore \underline{\underline{B=2}}$

$x=0 \quad 1 = -2A+2B-C$ 5

$-2A-C+4=1$

$2A+C=3$

x^3 සංගුණක සැසඳීම

$A-C=0$ 5

$A=C$

$A=C=1$ //

$2x^2+3x+1 = (x-1)(x^2+2) + 2(x^2+2) - (x+1)(x-1)^2$

$$\frac{2x^2+3x+1}{(x-1)^2(x^2+2)} = \frac{1}{x-1} + \frac{2}{(x-1)^2} - \frac{x+1}{x^2+2}$$ 10

$$\int \frac{2x^2+3x+1}{(x-1)^2(x^2+2)}\,dx = \int \frac{1}{x-1}\,dx + 2\int \frac{1}{(x-1)^2}\,dx - \int \left(\frac{x}{x^2+2} + \frac{1}{x^2+2}\right) dx$$ 5

$$= \ln|x-1| - \frac{2}{x-1} - \frac{1}{2}\ln(x^2+2) - \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + C$$

5 5 5 5

$\boxed{50}$

(b) $\int_0^{\pi/3} \sec x \, dx = \int_0^{\pi/3} \frac{\sec x (\sec x + \tan x)}{\sec x + \tan x} dx$ 5

$= \left[\ln |\sec x + \tan x| \right]_0^{\pi/3}$ 5

$= \ln \left| \frac{\sec \frac{\pi}{3} + \tan \pi/3}{\sec 0 + \tan 0} \right|$

$= \ln \left| \frac{2 + \sqrt{3}}{1 + 0} \right|$ 5

$= \ln (2 + \sqrt{3})$ //

$\int_0^{\frac{\pi}{3}} \sec^3 x \, dx = \int_0^{\pi/3} \sec x \cdot \sec^2 x \, dx.$

$= \left[\sec x . \tan x \right]_0^{\pi/3} - \int_0^{\pi/3} \tan x \cdot \sec x . \tan x \, dx$ 5 5

$= \left[\sec \frac{\pi}{3} . \tan \frac{\pi}{3} - 0 \right] - \int_0^{\pi/3} \sec x (\sec^2 x - 1) dx$

$= 2 \times \sqrt{3} - \int_0^{\pi/3} \sec^3 x \, dx + \int_0^{\pi/3} \sec x \, dx$ 5 5

$\int_0^{\pi/3} \sec^3 x \, dx = \frac{1}{2} \left[2 \times \sqrt{3} + \ln (2 + \sqrt{3}) \right]$

$= \sqrt{3} + \frac{1}{2} \ln (2 + \sqrt{3})$ //

5

$\boxed{40}$

$$I = \int_2^4 \sqrt{x^2-4}\ dx$$

$x = 2\sec\theta$ ලෙස ගනිමු.

$$\frac{dx}{d\theta} = 2\sec\theta \cdot \tan\theta$$

$x = 2 \qquad \sec\theta = 1 \ , \quad \theta = 0$

$x = 4 \qquad \sec\theta = 2 \ , \quad \theta = \frac{\pi}{3}$ 5

$$I = \int_0^{\pi/3} \sqrt{4\sec^2\theta - 4} \cdot 2\sec\theta \cdot \tan\theta \cdot d\theta$$ 5

$$= 4\int_0^{\pi/3} \sqrt{\sec^2\theta - 1} \cdot \sec\theta \cdot \tan\theta\ d\theta$$

$$= 4\int_0^{\pi/3} \sec\theta \cdot \tan^2\theta \cdot d\theta$$

$$= 4\int_0^{\pi/3} \sec\theta\ [\sec^2\theta - 1]\, d\theta$$

$$= 4\int_0^{\pi/3} \sec^3\theta - \sec\theta \quad d\theta$$ 5

$$= 4\left[\int_0^{\pi/3} \sec^3\theta \cdot d\theta - \int_0^{\pi/3} \sec\theta\ d\theta\right]$$

$$= 4\left[\sqrt{3} + \frac{1}{2}\tan(2+\sqrt{3}) - \ln(2+\sqrt{3})\right.$$

5

$$= 2\left[2\sqrt{3} - \ln(2+\sqrt{3})\right] //$$

$$I = \int_0^{\pi/2} \frac{x \sin 2x}{2 - \sin^2 2x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{(\frac{\pi}{2} - x)\sin(\pi - 2x)}{2 - \sin^2(\pi - 2x)}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{(\frac{\pi}{2} - x)\sin 2x}{2 - \sin^2 2x}\, dx \quad 5$$

$$2I = \int_0^{\pi/2} \frac{(\frac{\pi}{2} - x)\sin 2x + x \sin 2x}{2 - \sin^2 2x}\, dx \quad 5$$

$$= \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin 2x}{1 + \cos^2 2x}\, dx \quad 5$$

$$I = \frac{\pi}{4} \int_0^{\pi/2} \frac{-\frac{1}{2}\, d(\cos 2x)}{1 + \cos^2 2x}$$

$$= \frac{-\pi}{8}\left[\tan^{-1}(-1) - \tan^{-1}(1)\right] \quad 5$$

$$= -\frac{\pi}{8}\left[-\frac{\pi}{4} - \frac{\pi}{4}\right]$$

5

$$= \frac{\pi^2}{16} \,//$$

$\boxed{60}$

$$I = -\frac{\pi}{8}\left[\tan^{-1}(\cos x)\right]_0^{\frac{\pi}{2}} \; 10 \; = -\frac{\pi}{8}\left[\tan^{-1}\cos\pi - \tan^{-1}\cos 0\right]$$

$$= -\frac{\pi}{8}\left[\tan^{-1}(-1) - \tan^{-1} 1\right] = -\frac{\pi}{8}\left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = \frac{\pi^2}{16}$$

5

$$I = \int_2^4 \sqrt{x^2 - 4}\ dx$$

$x = 2\sec\theta$ ලෙස ගනිමු.

$$\frac{dx}{d\theta} = 2\sec\theta \cdot \tan\theta$$

$x = 2$ $\quad \sec\theta = 1 \ , \ \theta = 0$ 5

$x = 4$ $\quad \sec\theta = 2 \ , \ \theta = \frac{\pi}{3}$

$$I = \int_0^{\pi/3} \sqrt{4\sec^2\theta - 4} \cdot 2\sec\theta \cdot \tan\theta \cdot d\theta$$ 5

$$= 4\int_0^{\pi/3} \sqrt{\sec^2\theta - 1} \cdot \sec\theta \cdot \tan\theta\ d\theta$$

$$= 4\int_0^{\pi/3} \sec\theta \cdot \tan^2\theta \cdot d\theta$$

$$= 4\int_0^{\pi/3} \sec\theta\ [\sec^2\theta - 1]\ d\theta$$

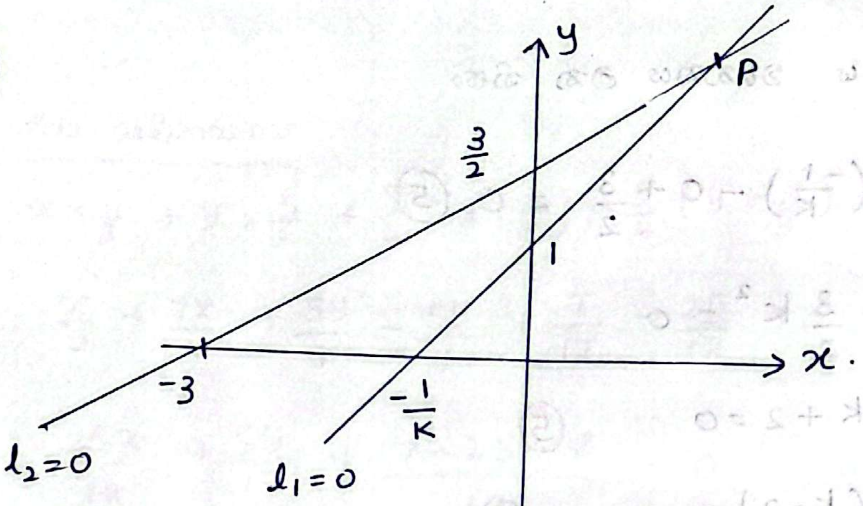
$$= 4\int_0^{\pi/3} \sec^3\theta - \sec\theta\ \ d\theta$$ 5

$$= 4\left[\int_0^{\pi/3} \sec^3\theta \cdot d\theta - \int_0^{\pi/3} \sec\theta\ d\theta\right]$$

$$= 4\left[\sqrt{3} + \frac{1}{2}\tan(2+\sqrt{3}) - \ln(2+\sqrt{3})\right]$$ 5

$$= 2\left[2\sqrt{3} - \ln(2+\sqrt{3})\right] \ //$$

(16)



වෘත්තයේ සමීකරණය $x^2+y^2+2gx+2fy+c=0$ 5 ලෙස ගනිමු.

$(-3,0)$, $(0,1)$ හා $\left(0,\frac{3}{2}\right)$ ලක්ෂ $S=0$ මත නිසා, 15

$$9+0+2g(-3)+0+c=0 \quad 5$$

$$6g-c = 9 \text{ — } (1)$$

$$0+1+0+2f\times 1+c=0 \quad 5$$

$$2f+c=-1 \text{ — } (2)$$

$$0+\frac{9}{4}+2g\times 0+2f\times\frac{3}{2}+c=0 \quad 5$$

$$3f+c=-\frac{9}{4} \text{ — } (3)$$

(3) − (2) න් $\quad f=-\frac{9}{4}+1=-\frac{5}{4}$ // 5

(2) $\Rightarrow \quad c=-1+\frac{5}{2}=\frac{3}{2}$ 5

(1) $\Rightarrow \quad 6g=9+\frac{3}{2}=\frac{21}{2}$;

$$g=\frac{7}{4} \quad 5$$

වෘත්තයේ සමීකරණය

$$x^2+y^2+\frac{7}{2}x-\frac{5y}{2}+\frac{3}{2}=0 \quad 5$$

$\left(-\frac{1}{k}, 0\right)$ ලක්ෂ්‍ය වෘත්තය මත නිසා

$$\frac{1}{k^2} + 0 + \frac{7}{2}\left(\frac{-1}{k}\right) - 0 + \frac{3}{2} = 0 \quad 5$$

$$1 - \frac{7k}{2} + \frac{3}{2}k^2 = 0$$

$$3k^2 - 7k + 2 = 0 \quad 5$$

$$(3k-1)(k-2) = 0 \quad 5$$

$$k = \frac{1}{3} \text{ හෝ } k = 2 \quad 5$$

$$k \in \mathbb{Z} \text{ නිසා } \underline{\underline{k = 2}} \quad 5$$

$\boxed{80}$

$k = 2$ විට,

$$l_1 \Rightarrow 2x - y + 1 = 0 \quad \text{—} (1) \quad 5$$

$$x - 2y + 3 = 0 \quad \text{—} (2)$$

(1) හා (2) විසඳීමෙන්,

$$x = \frac{1}{3}, \quad y = \frac{5}{3}$$

$$P \equiv \left(\frac{1}{3}, \frac{5}{3}\right) \quad 10$$

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AB සමීකරණය

$$x\times\frac{1}{3}+y\times\frac{5}{3}+\frac{7}{4}\left(x+\frac{1}{3}\right)-\frac{5}{4}\left(y+\frac{5}{3}\right)+\frac{3}{2}=0$$ (10)

$$\frac{x}{3}+\frac{7x}{4}+\frac{5y}{3}-\frac{5y}{4}+\frac{7}{12}-\frac{25}{12}+\frac{3}{2}=0$$

$$\frac{25x}{12}+\frac{5y}{12}+\frac{7-25+18}{12}=0$$

$$25x+5y=0$$

$$y+5x=0$$ (5)

A හා B හරහා යන ඕනෑම වෘත්තයක සමීකරණය

$$x^2+y^2+\frac{7}{2}x-\frac{5}{2}y+\frac{3}{2}+\lambda(5x+y)=0$$ (10)

$$x^2+y^2+\left(\frac{7}{2}+5\lambda\right)x+y\left(\lambda-\frac{5}{2}\right)+\frac{3}{2}=0$$

මෙහි කේන්ද්‍රය $=\left(\dfrac{5\lambda+\frac{7}{2}}{-2},\ \dfrac{\lambda-\frac{5}{2}}{-2}\right)$ (10)

මෙම ලක්ෂ්‍යය AB මත නිසා,

$$\frac{\lambda-\frac{5}{2}}{-2}+5\left(\frac{5\lambda+\frac{7}{2}}{-2}\right)=0$$ (10)

$$\lambda-\frac{5}{2}+25\lambda+\frac{35}{2}=0$$

$$26\lambda+\frac{30}{2}=0$$

$$26\lambda=-15$$

$$\lambda=\frac{-15}{26}$$ (5)

$\boxed{70}$

වෘත්තයේ සමීකරණය

$$x^2+y^2+\frac{7}{2}x-\frac{5}{2}y+\frac{3}{2}-\frac{15}{26}(5x+y)=0$$ (5)

17 (a) $\sin 2\theta = 2\sin\theta \cdot \cos\theta$ 5

$\cos 2\theta = \sin\left(\frac{\pi}{2} - 2\theta\right)$ 5

$= 2\sin\left(\frac{\pi}{4} - \theta\right) \cdot \cos\left(\frac{\pi}{4} - \theta\right)$

$= 2\left[\sin\frac{\pi}{4} \cdot \cos\theta - \cos\frac{\pi}{4} \cdot \sin\theta\right]\left[\cos\frac{\pi}{4} \cdot \cos\theta + \sin\frac{\pi}{4} \cdot \sin\theta\right]$ 5 5

$= 2\left[\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right]\left[\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right]$

$= (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

$= \cos^2\theta - \sin^2\theta$ 5

$\boxed{25}$

$f(x) = 4\sin x \cdot \sin 2x + 2\cos 2x + 2\cos x - 3$

$= 4\sin x \cdot 2\sin x \cdot \cos x + 2(\cos^2 x - \sin^2 x) + 2\cos x - 3$ 5

$= 8\sin^2 x \cdot \cos x + 2[1 - 2\sin^2 x] + 2\cos x - 3$ 5

$= 4\sin^2 x\,(2\cos x - 1) + 2\cos x - 1$

$= (4\sin^2 x + 1)(2\cos x - 1)$ 5

$a = 2$, $b = -1$, $c = 4$, $d = 1$ 5

$f(x) = (a\cos x + b)(4\sin^2 x + d)$ //

$\boxed{20}$

$f(x) = 2(2\cos x - 1)$

$(2\cos x - 1)(4\sin^2 x + 1) - 2(2\cos x - 1) = 0$

$(2\cos x - 1)(4\sin^2 x + 1 - 2) = 0$

$(2\cos x - 1)(4\sin^2 x - 1) = 0$ 5

$\cos x = \frac{1}{2}$ හෝ $\sin^2 x = \frac{1}{4}$ 5

$\sin x = \pm \frac{1}{2}$

$\cos x = \cos\frac{\pi}{3}$

$x = 2n\pi \pm \frac{\pi}{3}\ ;\ n \in \mathbb{Z}$ 5

$\sin x = \frac{1}{2} = \sin\frac{\pi}{6}$

5 $x = n\pi + (-1)^n \frac{\pi}{6}\ ;\ n \in \mathbb{Z}$

$\sin x = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$

5 $x = n\pi - (-1)^n \frac{\pi}{6}\ ;\ n \in \mathbb{Z}$

$\boxed{25}$

(b) සම්මත අංකනයට අනුව ABC ත්‍රිකෝණයේ

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ වේ 5

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k}$ ලෙස ගනිමු.

$(b-c)\cot\frac{A}{2} = (k\sin B - k\sin C)\cot\frac{A}{2}$ 5

$= k\left(2\cos\frac{B+C}{2} \cdot \sin\frac{B-C}{2}\right)\cot\frac{A}{2}$ 5

$= 2k\cos\left(\frac{\pi}{2} - \frac{A}{2}\right) \cdot \sin\left(\frac{B-C}{2}\right) \cdot \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}$ 5

$= 2k\sin\left(\frac{B-C}{2}\right) \cdot \cos\left(\frac{\pi}{2} - \frac{B+C}{2}\right)$

$= 2k\sin\left(\frac{B-C}{2}\right) \cdot \sin\left(\frac{B+C}{2}\right)$ 5

$$(b-c)\cot\frac{A}{2} = -k\,[\cos B - \cos C]$$
$$= k\,[\cos C - \cos B] \quad —① \quad (5)$$

එලෙසම,

$$(c-a)\cot\left(\frac{B}{2}\right) = k(\cos A - \cos C) \quad —② \quad (5)$$
$$(a-b)\cot\left(\frac{C}{2}\right) = k(\cos B - \cos A) \quad —③ \quad (5)$$

① + ② + ③

$$(b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2} = 0 \; // \quad (5)$$

$\boxed{45}$

(6) $\tan^{-1}\left(\dfrac{x-1}{2x-1}\right) + \tan^{-1}\left(\dfrac{x+1}{2x+1}\right) = \dfrac{\pi}{4}$

$A = \tan^{-1}\left(\dfrac{x-1}{2x-1}\right)$ හා $B = \tan^{-1}\dfrac{x+1}{2x+1}$ ලෙස ගනිමු.

එවිට $-\dfrac{\pi}{2} < A, B < \dfrac{\pi}{2}$ වේ.

$\tan A = \dfrac{x-1}{2x-1}$ (5), $\tan B = \dfrac{x+1}{2x+1}$ (5)

$$A + B = \frac{\pi}{4}$$
$$\tan(A+B) = \tan\frac{\pi}{4} \quad (5)$$
$$(5) \quad \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$$
$$\frac{\dfrac{x-1}{2x-1} + \dfrac{x+1}{2x+1}}{1 - \dfrac{x-1}{2x-1} \times \dfrac{x+1}{2x+1}} = 1 \quad (5)$$
$$\frac{(x-1)(2x+1) + (x+1)(2x-1)}{(2x-1)(2x+1) - (x^2-1)} = 1$$
$$\frac{4x^2-2}{3x^2} = 1 \implies x^2 = 2 \quad (10)$$
$$x = \pm\sqrt{2}$$

$\boxed{35}$